

XXVIII. *A Letter to the Right Honourable the Earl of Macclesfield, President of the Royal Society, concerning the Method of constructing a Table for the Probabilities of Life at London, from the Reverend William Brakenridge, D. D. and F. R. S.*

My Lord,

Read April 24, 1755.

YOUR character in the philosophical world, in the relation you bear to each member of our illustrious Society, makes me presume to offer my thoughts to you, on a very interesting subject, the probability of human life from the bills of mortality. For as it has some difficulty, and requires an accurate examination of many circumstances, I could not possibly send such calculations to any one who understood them better, or could more quickly discover any mistake, or fallacious reasoning; and I hope your usual goodness, and indulgence to every industrious inquirer will excuse me, that I give you this trouble.

The great Dr. Halley, who had a singular faculty of applying his mathematical knowledge to the purposes of life, was the first who particularly attended to this subject. In the year 1692, from the bills of mortality at Breslau, he reduced it into a sort of science; and gave a table of the probabilities of life, that hitherto has been justly esteemed the most exact of any thing of the kind; from which he and others have deduced many propositions, that are

highly useful. But a doubt having arisen, whether that could properly, or with any accuracy, be used by us at London, as we are in a different country, and perhaps in a different way of life, I have been at some pains to inquire into this, and satisfy myself about the objections. And I imagine, that I can now shew how that table may be altered, to suit our case with sufficient exactness.

In the London bills of mortality, for the last 30 years, there is always added an account yearly of the number of burials under each age, at the distance of ten years, and of children more particularly under two years, between two and five, and between five and ten; which numbers are curious and useful. And, I believe, though there may sometimes be some inaccuracies and omissions, these numbers are as exactly given as in our case can be expected: and what may be objected, is not so much to the incorrectness of them, but to what arises from our circumstances, that will not allow them to be proper, to shew the probabilities of life in all its periods.

As I am inclined to think, that no table can be formed from them, as they are at present, that will be sufficiently accurate above 20 years of age, I shall make some observations that seem to shew this, before I proceed to what I have proposed. And that we may have a more accurate view of the numbers, and be able more certainly to reason about them, let us take the sums of all the burials under each age, for the last ten years, from 1744 to 1753 inclusive; and then the tenth part of those sums will give, at an average, the burials at each particular age in a year: And they will stand as in the 2d column of the annexed table.

table. Of which the sum for one year is 22867. But if we suppose, that there are 2000 more burials yearly than in the bills, on account of dissenters of all denominations, and those burials that are carried out of town, and not accounted for, as has been mentioned in my letter last year, concerning the number of inhabitants; the whole number of burials will then be 24867 yearly. And we must take the proportional parts of 2000, and add them to the numbers of burials at the different ages, which will give all the burials at each; and the numbers so augmented will stand as in the 3d column. And if we likewise take the numbers of the dead, at the same periods at Breslau, they will be as in the 4th column. The numbers in the first denote the years.

2	8110	8819	202
2—5	1845	2006	88
5—10	741	805	57
10—20	682	741	61
20—30	1904	2070	69
30—40	2301	2502	87
40—50	2395	2604	101
50—60	1881	2045	103
60—70	1481	1610	101
70—80	1053	1145	97
80—90	474	515	33
Sums	22867	24862	999

Now when we consider the series of numbers, representing the dead at London, it seems plain, that above 20 years of age it is unnatural; and therefore

cannot be a true representation of the probability of life. In the age between 20 and 30 there appears to die near three times the number, that die between 10 and 20. For the numbers are as 2070 to 741; which cannot possibly be from the natural decrements of life; because we generally see, that people between 20 and 30 are nearly as healthy and vigorous, as between 10 and 20; but it must necessarily arise, from the great increase of the number of the people at that age, by the accession of strangers. And then, on the contrary, above the age of 50, the numbers seem to represent the decrements of life slower, contrary to the natural course: For the numbers of the dead after that age continually decrease. From 40 years of age to 50 they are 2604, but in the three following decennial periods they are as 2045, 1610, 1145; that is, in the more healthy ages there are more burials, than in the more infirm and unhealthy; which is not to be accounted for without supposing, that great numbers retire after the age of 50 into the country. For after that time, as infirmities and diseases greatly increase, the burials ought at least not to be fewer, till the number of people is nearly exhausted. And indeed it is evident to common observation, that a great number in advanced age retire from the town; from which it must follow, that the numbers of the dead cannot give a true representation of the probability of life.

If, in the next place, we compare the numbers of the dead, in the several periods at Breslau, with those at London, we shall plainly see, that the former shew the decrements of life in a natural and regular way, and free from the above difficulties and objections.

tions. In the infant state, under 2 years of age, there is one-fifth lost by death; but afterwards, as they gather strength, the deaths are diminished till between 10 and 20; and from that age the mortality gradually increases, till after the age of 40; when the number of the dead continues nearly the same, though the probability of life continually decreases till the age of 80; and then at length, the living being almost all exhausted, the burials are greatly diminished. All which seems to be agreeable to the course of nature; but, contrary to what we see in the London bills, especially after 50 years of age, as we have already observed. However, they both agree in this, that the most healthy age is between 10 and 20, and the infant state under 5 years of age the most uncertain for life.

But indeed it must be acknowledg'd, that in computing the Breslau table, Dr. Halley had great advantages, which have made it so perfect. He had the number of births given, besides the burials at the different ages, in an inland town, where there is no great concourse of strangers. But with us at London the number of births is not known; because of the number of dissenters of various denominations, both foreigners and natives, of whose baptisms there is no account taken; which makes our bills at present very imperfect. For none are put into our bills but those who are baptized, according to the form of our established church. And therefore there are some thousands omitted, and yet many, perhaps the one-half of them, who are not baptized with us, bury with us; which greatly perplexes our bills. And under this disadvantage it appears very

difficult, to make an accurate computation of the decrements of life through the different ages ; though this defect I imagine I shall be able nearly to supply.

But I think, without examining the series of numbers resulting from our bills, if we only consider the circumstances of our great Metropolis, we shall plainly see, that above 20 years of age there can be no depending on the number of our burials, so as to compute the probability of life. For after that age, there is such a multitude of strangers come here, from all places ; some to settle for life, others to stay for a time to acquire some fortune or employment, and others only to remain a few years or months, that our burials are always in a fluctuating state. Servants, journey-workmen, and young people, that are to push into life, generally come at that age ; some of which come only for two or three years ; and then, if they survive, retire again into the county ; all which occasions such a great number of accidental burials, that it seems plain, that our bills cannot be so regular as to shew the probability of life above 20 years of age. And therefore, by the way, it may be observed, that the greatest number of people is between 20 and 30 ; for at that age there is above 160000, that is, above a fifth of the whole ; which is twice the number that is between 10 and 20. And thus, I think there are so great objections to our bills, in the ages above 20, that they seem to me to be improper, to make any computation of the value of lives from them. And it were to be wished, that some inland town could be found in England, where there was kept an accurate register of births and burials, with the ages of the deceased, and where there

there is no great confluence of strangers; or rather that a dozen or twenty parishes, contiguous to one another in the country, could be found to keep such a register; for that would be of more general use.

There have indeed been some ingenious men who have thought, that our London bills are correct enough to form a table from them, which may better agree with our circumstances, than that which Dr. Halley has given us. And Mr. Smart was the first who endeavoured to do something in this way, from our bills only, about 18 years ago. But, in the table made by him, he seems to have been greatly mistaken; for he has made no allowance for the accession of strangers, but considered the numbers of the dead, in all the periods of life, as all come from those born here; whereas it is evident, that the strangers, above 20 years of age, are at least equal to them. And this has brought this paradox into his table, that young people between 12 and 18, at London, are much more healthy than at Breslau, or in any country place in England. For according to him, in the 13th year, 2 die only out of 479; but at Breslau there die 6 out of 634; that is, there is double the number die more at Breslau than at London; which appears impossible. But between 30 and 40, he makes them much more unhealthy than they are; for at 40 he supposes one to die in 29; whereas there does not die above one in 30, all ages taken together, with infants included, as I have shewn in my letter last year. Another ingenious gentleman, having seen this inconsistency, has endeavoured to correct it, by supposing that the number of strangers that come to settle in town, after 25 years

of age, is inconsiderable ; and that above that age, the numbers of burials may be considered, as arising from the natural degrees of mortality ; and then by proportion, increasing the numbers of the living corresponding to all ages below 25 ; so that the table, altered in this manner, is the same with Mr. Smart's above that age. And it must be confessed, that this correction is very proper, and worthy of its author. But still the table is greatly defective, as he has made no allowance for the recess of great numbers, who after they have been a number of years in town, leave it, if they survive ; and of many others who, after the age of 50, retire from business into the country. And which is so very obvious, that our burials are fewer, than by proportion they ought to be after 50 years of age, as I have mentioned above, and by consequence the people appear more healthy after that age ; so that after 70 they seem more healthy than at Breslau. For at 75 there appears from this table to die 4 out of 45, whereas at Breslau there die 10 out of 88.

And that a great number retire from the town, after the age of 50, or before, is farther evident ; if we suppose, even according to this corrected table, that one in 25 die at the age of 50. For then the number of people alive, between 40 and 50, will be greater than 2604 multiplied by 25, or 65100 ; which ought to be exhausted by all the deaths in the subsequent periods. But all the deaths which ought to arise from that number of living, in the following years to 90, according to the bills in the 3d column, is 5315 multiplied by 10, or 53150 ; which is less than the people that were alive between 40 and 50,

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by 11950, or more. And therefore above 11000 of those between 40 and 50, must have retired from town.

But now, as our bills are defective, it is next to be considered, what we at London are to do at present, and what method of computation we are to follow? And I imagine it is very obvious what may be done. Our bills may be used so far as 14 or 20 years; for there is certainly no increase of our people till the age of 14; because few young people come to town till they are fit to be apprentices or servants. And between 14 and 20, though many come at that time, yet there is an emigration of a great number from hence to sea, to other countries, the universities, and country academies, that nearly balances the accession of strangers. And then, after 20 years of age, I believe the Breslau bills will be sufficiently correct, to shew the probability of life within and about the city. And if so, a table may be made from both bills, that will agree with our case here with sufficient exactness. For I cannot find, that there is any difference in the bills, above the age of 20, that can be depended upon. And I see no reason why our air should not agree, as well with those in advanced life, as that at Breslau, and our people be equally healthy. And this is not mere conjecture; for, as far as I can examine into it, a variety of things seem to confirm it.

Breslau is nearly in the same parallel of latitude with London, and therefore their seasons cannot be very different; and it is a large town, containing above thirty thousand inhabitants, according to Dr. Halley; and so the air, and circumstances of life, can-

not

not be so conducive to health as in the open country. And consequently unless all great towns that are less than London, and nearly in the same latitude, are more healthy, there can no reason be given why Breslau should be so. It is true it cannot be demonstrated, that they are equally healthy, to persons above 14 or 20 years of age; but when we see the tables that have been made from our bills, and observe the inconsistency that appears in them, which is owing to the fluctuating state of our city; we see no reason to think otherwise, and we rather seem to have some presumption that it is so. Because, if we look into Mr. Smart's table, we find, that he has made the people more healthy between 12 and 18, and after 70 years of age, than at Breslau; and yet more unhealthy in the ages between these periods; which seems to be near to a contradiction: For why they should be more unhealthy in the intermediate years, is not possible to be accounted for. One would rather think, that if in the extremities of life they were more healthy at London, they could not be less healthy in the middle, and stronger part of it. Or, if we consult another form of this table ingeniously corrected, we see, that people appear still more healthy after 70 than at Breslau; which seems to be inconsistent with their being more unhealthy in younger life. For one would imagine, that the inclemency of any climate or place should affect people more with the infirmities of age, than those that are younger, and yet in their vigour. But if we account for this variation of the table, by supposing, that after the age of 50, many retire from town, as I have mentioned

tioned above, this will destroy the whole hypothesis, upon which the table is formed.

If we suppose that one in 34 die at Breslau in the year, as appears from Dr. Halley's table, and one in 30 at London, this difference does not shew, that the probabilities of life in both places are not the same above 20 years of age; for it may be fully accounted for, from the different probabilities under the age of 4. Because every year, at London, there dies one-fourth of the infants, under 4 years of age, more than at Breslau; which, without considering the other differences that may be between that age and 15, will more than account for the difference I have mentioned. At London the proportion of one in 30, comprehends all, as well strangers as natives; but at Breslau the proportion of one to 34 is only to be understood of those that are born there. Mr. Smart, in his table, has made the proportion among those that are born at London to be one to 24, and in his table corrected it is made as one to 19; but in the table that I have constructed from the Breslau and London together, it is one to 21.5; which is a medium between them.

If it be said, that from our bills the infant state, under the fifth year, certainly appears more unhealthy than at Breslau, and therefore probably it is so in advanced life: It may be answered, that possibly our gross air may not suit so well with infants, and yet may agree well enough with them after they become stronger; as we see some sorts of food are improper for the infant state, and yet do very well when nature comes to maturity. And it may likewise be alleged, that perhaps we are in a bad way of

managing of them, that many are destroyed, with sleepy and poisonous cordials, and others lost thro' the want of care and tenderness, or wickedness of our parish-nurses ; for I know that there is not one in five survives their management.

And thus, from all these considerations, I think it may be allowed, until it is otherwise demonstrated, by bills formed in a different manner from what we have at present, that the probabilities of life are much the same at London as at Breslau, at the age of 20 or after 14. And if we take this for granted, we shall from thence be able to form a useful table, for those within our bills, by accommodating and joining the bills of both places together. And we may also nearly determine the number of infants born here, which hitherto has not been considered.

To find the number of births, by which I mean all those that are born alive, so as they might have been baptized, we must have the number of burials known, at least in the several periods, till the 20th year ; *viz.* under 2, between 2 and 5, between 5 and 10, and between 10 and 20. And it is evident, if we suppose no accession of strangers, that the number of living in any one year will be equal to the difference between the births, and the sum of all the subsequent burials at each age till that year. The number of the living in any one year is easily known, if we suppose the probability of life to be the same as at Breslau ; for then the number of dead there will be to the number of living, as the dead at London to the living. Thus in the 20th year the dead and living at Breslau are as 6 and 598, and the dead at London are 73, or more exactly 72, 88 ; therefore

the living must be 7263. The dead in the intermediate years at London may likewise be found, by means of Dr. Halley's table. For, by proportion, if the dead at Breslau from the age of 10 to 20 complete be 61, and in the 20th year 6, and the dead at London for the same period be 741; then will the dead in the 20th year be 73. And therefore if the living at London, in the 20th year of their age, be found to be 7263; this must be equal to the number of births, having subtracted from them all the dead in each of the preceding nineteen years. And consequently if we put x for the number of births, we shall have this simple equation,

$x - 8819 - 2006 - 805 - 741 + 73 = 7263$; and thence the number of births $x = 19561$. And the same number would have been produced from any intermediate age, between 12 and 20. So that if we could be certain of the number of the dead, there could be no doubt but, that 19561 would nearly at an average for ten years, be the whole of the births yearly. And this is greater than the number of baptisms known 14626, taken likewise at a medium, for the same ten years, from 1743 to 1753 inclusive, by the number 4935.

From which, by the way, we may see, as this difference between the births and baptisms must be occasioned by dissenters, that the number of such of all denominations, both protestant and popish, with the Jews, do not make above one-fourth of the whole of the people within the bills of mortality; and consequently that the protestant dissenters, exclusive of Quakers and Jews, are not above an eighth part of the whole. And we may also observe, that as the difference between the births

19561, and burials 24867 is 5306, there must be a constant supply, yearly, of at least 5000 strangers, to keep up the people within the bills, to their present number: And the births are to the dead yearly, about 4 to 5.

If we had, in the same manner, computed the births from Mr. Smart's corrected table, they would have been found to be 17992, that is, 1569 fewer than we have made them. And if from thence we had calculated the number of people living to 20 years of age, and afterwards, by proportion, to 90, the whole number of people within the bills would have been about 521000; which is above 150000 fewer than any other reasonable calculation can make them; which I think clearly shews, that the hypothesis upon which that table is founded must be wrong, and that what I have laid down above is nearer to the truth.

Now, from the births found 19561, and the numbers of the dead in the different periods known by our bills, it will be easy to form a table of the decrements of life; because the dead in the intermediate years may be found by what has been said above. And accordingly I have computed the following, which is constructed from the London and Breslau bills together; which I think is a surer method of computing for us at London, than from either of them alone. The first part to the 21st year, is done from our bills, and the other part from the Breslau; but it is formed in such a manner, that it goes on as if from the bills of one place only. For, after the age of 20, it is continued by proportion, by making the dead at London in the decennial periods, to have the same ratio to one another as the dead at Breslau.

Breslau. It supposes 1000 persons born in one year, and shews the annual decrease of them by death till 87 years of age, which may be considered as the utmost period of life. The intermediate numbers, marked *d*, shew the dead in each year. The use of this table is well known to all who can compute the value of annuities for lives.

Age. Perf.	Age. Perf.	Age. Perf.	Age. Perf.
1 1000	14 394	27 344	40 277
323 d	4 d	4 d	6 d
2 677	15 390	28 340	41 271
127 d	4 d	5 d	6 d
3 550	16 386	29 335	42 265
45 d	3 d	5 d	6 d
4 505	17 383	30 330	43 259
32 d	4 d	5 d	6 d
5 473	18 379	31 325	44 253
26 d	4 d	5 d	6 d
6 447	19 375	32 320	45 247
13 d	3 d	5 d	6 d
7 434	20 372	33 315	46 241
9 d	4 d	5 d	6 d
8 425	21 368	34 310	47 235
7 d	4 d	5 d	6 d
9 418	22 364	35 305	48 229
6 d	4 d	6 d	7 d
10 412	23 360	36 299	49 222
6 d	4 d	6 d	7 d
11 406	24 356	37 293	50 215
4 d	4 d	5 d	7 d
12 402	25 352	38 288	51 208
4 d	4 d	6 d	7 d
13 398	26 348	39 282	52 201
4 d	4 d	5 d	7 d

Age.	Perf.	Age.	Perf.	Age.	Perf.	Age.	Perf.
53	194	62	139	71	82	80	26
	7 d		7 d		7 d		4 d
54	187	63	132	72	75	81	22
	6 d		6 d		7 d		4 d
55	181	64	126	73	68	82	18
	6 d		6 d		7 d		3 d
56	175	65	120	74	61	83	15
	6 d		6 d		6 d		2 d
57	169	66	114	75	55	84	13
	6 d		6 d		6 d		2 d
58	163	67	108	76	49	85	11
	6 d		7 d		6 d		2 d
59	157	68	101	77	43	86	9
	6 d		6 d		6 d		2 d
60	151	69	95	78	37	87	7
	6 d		6 d		6 d		
61	145	70	89	79	31		
	6 d		7 d		5 d		

And now, my Lord, I doubt I have made this Letter too long; but the importance of the subject will, I hope, excuse me. And if I have been mistaken in any particulars, I must confide in your usual goodness to forgive me; for you are sensible of the difficulties that occur. My endeavours, if I have not succeeded, will perhaps excite others, who have more leisure, and greater abilities, to make farther inquiries. I have some other things upon other subjects which I shall presume shortly to offer to your consideration. And in the mean time I am, with the greatest affection, My Lord,

Sion-College,
April 24, 1755.

Your Lordship's most devoted
and faithful servant,
W^m. Brakenridge.

Breslau. It supposes 1000 persons born in one year, and shews the annual decrease of them by death till 87 years of age, which may be considered as the utmost period of life. The intermediate numbers, marked *d*, shew the dead in each year. The use of this table is well known to all who can compute the value of annuities for lives.

Age.	Perf.	Age.	Perf.	Age.	Perf.	Age.	Perf.
1	1000	14	395	27	345	40	278
	323 d		4 d		4 d		6 d
2	677	15	391	28	341	41	272
	127 d		4 d		5 d		6 d
3	550	16	387	29	336	42	266
	45 d		3 d		5 d		6 d
4	505	17	384	30	331	43	260
	32 d		4 d		5 d		6 d
5	473	18	380	31	326	44	254
	26 d		4 d		5 d		6 d
6	447	19	376	32	321	45	248
	13 d		3 d		5 d		6 d
7	434	20	373	33	316	46	242
	9 d		4 d		5 d		6 d
8	425	21	369	34	311	47	236
	7 d		4 d		5 d		6 d
9	419	22	365	35	306	48	230
	6 d		4 d		6 d		7 d
10	413	23	361	36	300	49	223
	6 d		4 d		6 d		7 d
11	407	24	357	37	294	50	216
	4 d		4 d		5 d		7 d
12	403	25	353	38	289	51	209
	4 d		4 d		6 d		7 d
13	399	26	349	39	283	52	202
	4 d		4 d		5 d		7 d

Age.	Perf.	Age.	Perf.	Age.	Perf.	Age.	Perf.
53	195	62	130	71	73	80	17
	7 d		7 d		7 d		4 d
54	188	63	123	72	66	81	13
	6 d		6 d		7 d		4 d
55	182	64	117	73	59	82	9
	6 d		6 d		7 d		3 d
56	176	65	111	74	52	83	6
	6 d		6 d		6 d		2 d
57	170	66	105	75	46	84	4
	6 d		6 d		6 d		1 d
58	164	67	99	76	40	85	3
	6 d		7 d		6 d		1 d
59	158	68	92	77	34	86	2
	6 d		6 d		6 d		1 d
60	142	69	86	78	28	87	1
	6 d		6 d		6 d		
61	136	70	80	79	22		
	6 d		7 d		5 d		

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